محاضرات في هياكل متقطعة

قسم تربية الحاسبات كلية الطوسي

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المرحلة الأولى

Discrete Structure Introduction:

Discrete structures involve elements that are distinct and separate, often countable. Examples include integers, graphs, sets, sequences, and relations.

Why discrete structure:

Computers use discrete structures to represent and manipulate data.
Computer Science is not Programming Computer Science is not Software Engineering Computer Science is about problem solving.
Mathematics is at the heart of problem solving
Defining a problem requires mathematical accuracy
Use and analysis of models, data structures, algorithms requires a solid foundation of mathematics
To justify why a particular way of solving a problem is correct or efficient (i.e., better than another way) requires analysis with a well-defined mathematical model.

Discrete structures refer to mathematical structures that are distinct and separate, rather than continuous.
In summary, discrete structures provide a formal framework for understanding and solving problems in various domains in the fields of computer science, mathematics, and beyond.

Several reasons contribute to the importance of discrete structures:

Foundations of Computer Science:

- Algorithms and Data Structures: Discrete structures provide the foundation for designing and analyzing algorithms and data structures, which are essential components in computer science.
- Logic and Boolean Algebra: Discrete structures, such as propositional and predicate logic, form the basis for understanding and designing logical circuits in computer systems.

Computer Programming:

- Programming Concepts: Discrete structures help programmers understand fundamental concepts like sets, relations, functions, and graphs, which are widely used in programming and software development.
- Problem Solving: Discrete structures aid in developing problem-solving skills, as many real-world problems can be effectively represented and solved using discrete mathematics.

Cryptography and Security:

- Number Theory: Discrete structures like number theory play a crucial role in cryptography and information security. Concepts such as prime numbers, modular arithmetic, and encryption algorithms are based on discrete mathematical principles.

Artificial Intelligence:

- Graph Theory: Discrete structures like graph theory are fundamental in modeling and solving problems in artificial intelligence, network analysis, and optimization.
- Combinatorics: Combinatorial methods, a branch of discrete mathematics, are employed in various AI applications for search algorithms and problem-solving strategies.

Mathematical Modeling:

- Combinatorics and Probability: Discrete structures help model and analyze situations involving combinations, permutations, and probability, which are essential in statistical analysis and decision-making.
- Graphs and Networks: Graph theory is extensively used in modeling relationships and connections between entities, making it applicable in various scientific and social contexts.

Database Management:

- Relations and Databases: Discrete structures, particularly relations, are used in database management systems to organize and represent data effectively.

Information Theory:

- Coding Theory: Discrete structures, such as error-correcting codes, are integral in information theory and communication systems to ensure accurate transmission and storage of data.

Set Theory

Set theory is a foundational branch of mathematical logic that studies sets, which are collections of distinct objects. The objects in a set are called elements. Set theory provides a formal language for describing mathematical concepts and relationships, and it serves as the basis for much of modern mathematics.

Here are some key concepts and principles in set theory:

1. **Sets and Elements:**
   - A set is defined by listing its elements inside curly braces, such as $A = \{1, 2, 3\}$.
   - The symbol $\in$ denotes "belongs to," so $1 \in A$ means that 1 is an element of set A.
2. **Set Operations:**
   - **Union ($\cup$):** $A \cup B$ is the set of elements that are in A, or in B, or in both.
   - **Intersection ($\cap$):** $A \cap B$ is the set of elements that are in both A and B.
   - **Complement ($\complement$):** The complement of set A, denoted by A' or $\complement A$, is the set of elements not in A.

Example 1 Let $A = \{1, 3, 5, 7\}$. Then $1 \in A$, $3 \in A$, but $2 \notin A$.

Example 2 The set consisting of all the letters in the word "byte" can be denoted by {b, y, t, e} or by $\{x \mid x$ is a letter in the word "byte"$\}$. ◆

Example 3 We introduce here several sets and their notations that will be used throughout this book.

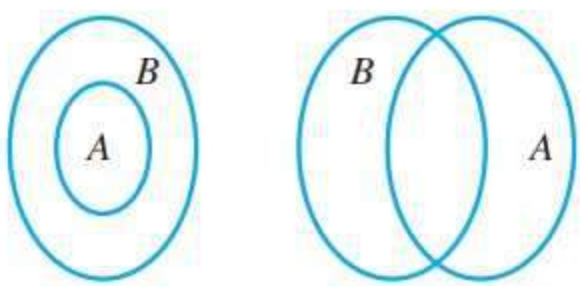
(a) $\mathbb{Z}^+ = \{x \mid x \text{ is a positive integer}\}$.
Thus $\mathbb{Z}^+$ consists of the numbers used for counting: $1, 2, 3 \ldots$.

(b) $\mathbb{N} = \{x \mid x \text{ is a positive integer or zero}\} = \{x \mid x \text{ is a natural number}\}$.
Thus $\mathbb{N}$ consists of the positive integers and zero: $0, 1, 2, \ldots$.

(c) $\mathbb{Z} = \{x \mid x \text{ is an integer}\}$.
Thus $\mathbb{Z}$ consists of all the integers: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$.

Subsets

In set theory, a subset is a set that contains only elements that are also elements of another set. If every element of set A is also an element of set B, then A is a subset of B, denoted as $A \subseteq B$.



$A \subseteq B$ $\qquad$ $A \nsubseteq B$

Diagrams, such as those in Figure above, which are used to show relationships between sets, are called Venn diagrams.

Example Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 5\}$, and $C = \{1, 2, 3, 4, 5\}$. Then $B \subseteq A$, $B \subseteq C$, and $C \subseteq A$. However, $A \nsubseteq B$, $A \nsubseteq C$, and $C \nsubseteq B$.

Example

Let's take an example involving the empty set:

$G = \{\}$

$H = \{1, 2, 3\}$

Here, G is a subset of H because every element in G is also in H, even though G is the empty set: $G \subseteq H$.

Example

Consider the universal set U and a set I:

$U = \{a, b, c, d, e\}$

$I = \{a, b, c\}$

In this case, I is a proper subset of U ($I \subset U$) because I is a subset of U but I is not equal to U.

Exercise

1. Let $A = \{1, 2, 4, a, b, c\}$. Identify each of the following as true or false.

   (a) $2 \in A$ **(b)** $3 \in A$ **(c)** $c \notin A$

   (e) $\{\ \} \notin A$ **(f)** $A \in A$

Solution:

1. $2 \in A$: **True** - The element 2 is in the set A.
2. $3 \in A$: **False** - The element 3 is not in the set A.
3. $c \notin A$: **True** - The element c is not in the set A.
4. $\{\} \notin A$: **True** - The empty set $\{\}$ is not an element of A.
5. $A \notin A$: **False** - Sets are not generally considered elements of themselves. Therefore, this statement is typically considered false in standard set theory. The set A is not an element of itself.

٢- In each part, give the set of letters in each word by listing the elements of the set. (a) **AARDVARK (b) BOOK (c) MISSISSIPPI Solution:**

(a) **AARDVARK:**

$A = \{A, R, D, V, K\}$

(b) **BOOK:**

$B = \{B, O, O, K\}$

(Note: In a set, each element is unique, so the repeated letter 'O' is listed only once.)

(c) **MISSISSIPPI:**

$M = \{M, I, S, P\}$

$I = \{I\}$

$S = \{S\}$

$P = \{P\}$

(Note: In a set, each element is unique, so the repeated letters 'I', 'S', and 'P' are listed only once in their respective sets.)

Each set represents the unique letters in the corresponding word.

٣- Let A = {١,{٢, ٣}, ٤}. Identify each of the following as true or false.

Solution:

This set consists of three elements: the number ١, the set {٢,٣}, and the number ٤.

(a) $3 \in A$: **False** - The element 3 is not directly in the set A, but it is an element of the set $\{2, 3\}$ within A.

(b) $\{1, 4\} \subseteq A$: **True** - Every element in the set $\{1, 4\}$ is also in A.

(c) $\{2, 3\} \subseteq A$: **True** - Every element in the set $\{2, 3\}$ is also in A, specifically within the set $\{2, 3\}$ in A.

(d) $\{2, 3\} \in A$: **True** - The set $\{2, 3\}$ is an element of A.

(e) $\{4\} \in A$: **True** - The set $\{4\}$ is an element of A.

(e) $\{4\} \in A$: **True** - The set $\{4\}$ is an element of A.

(f) $\{1, 2, 3\} \subseteq A$: **False** - While A contains the set $\{2, 3\}$, it does not directly contain the set $\{1, 2, 3\}$, so the statement is false.

Let $A = \{x \mid x \text{ is an integer and } x^2 < 16\}$. Identify each of the following as true or false.

(a) $\{0, 1, 2, 3\} \subseteq A$ **(b)** $\{-3, -2, -1\} \subseteq A$

(c) $\{\ \} \subseteq A$

(d) $\{x \mid x \text{ is an integer and } |x| < 4\} \subseteq A$

(e) $A \subseteq \{-3, -2, -1, 0, 1, 2, 3\}$

Solution:

The set A includes the integers -٣, -٢, -١, ٠, ١, ٢, and ٣.

(a) $\{0, 1, 2, 3\} \subseteq A$: **True** - Every element in the set $\{0, 1, 2, 3\}$ is also in A.

(b) $\{-3, -2, -1\} \subseteq A$: **True** - Every element in the set $\{-3, -2, -1\}$ is also in A.

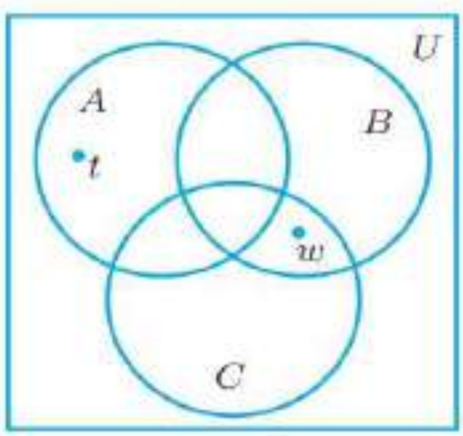
(c) $\{\} \subseteq A$: **True** - The empty set is a subset of every set, so this statement is true.

(d) $\{x \mid x \text{ is an integer and } |x| < 4\} \subseteq A$: **True** - The set of integers satisfying $|x| < 4$ includes all integers from -3 to 3, so it is a subset of A.

(e) $A \subseteq \{-3, -2, -1, 0, 1, 2, 3\}$: **True** - Every element in A is also in $\{-3, -2, -1, 0, 1, 2, 3\}$.

Use the Venn diagram in Figure 1.4 to identify each of the following as true or false.

(a) $B \subseteq A$ **(b)** $A \subseteq C$ **(c)** $C \subseteq B$
(d) $w \in A$ **(e)** $t \in A$ **(f)** $w \in B$



Operations on Sets

This section introduces a number of set operations, including the basic operations of **union, intersection, and complement. Example:**

Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{2, 3, 8, 9\}$. Then

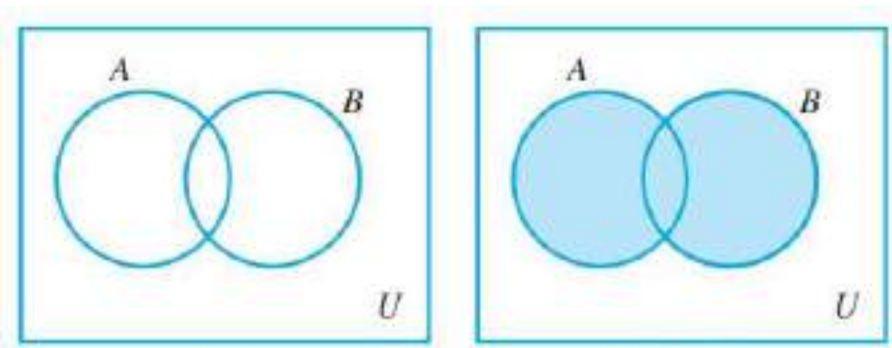
$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}, \quad A \cup C = \{1, 2, 3, 4, 8, 9\}, \quad B \cup C = \{2, 3, 4, 5, 6, 7, 8, 9\},$$
$$A \cap B = \{3, 4\}, \quad A \cap C = \{2, 3\}, \quad B \cap C = \{3\}.$$

Example:

Let A = {a, b, c, e, f } and B = {b, d,r,s}. Find A ∪ B and draw the venn diagram.

Solution

Since A ∪ B consists of all the elements that belong to either A or B, A ∪ B = {a, b, c, d, e, f,r,s}.



(a) (b) $A \cup B$

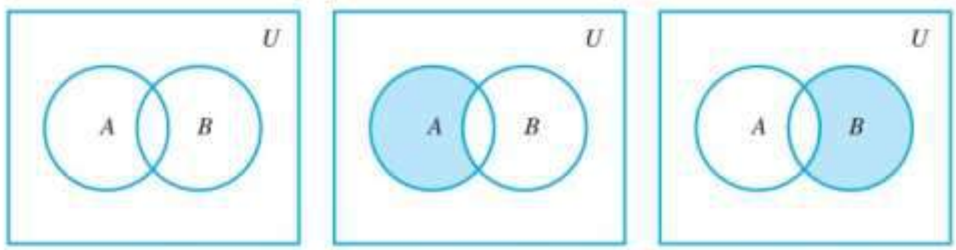
Example 2 Let $A = \{a, b, c, e, f\}$, $B = \{b, e, f, r, s\}$, and $C = \{a, t, u, v\}$. Find $A \cap B$, $A \cap C$, and $B \cap C$.

Solution

The elements b, e, and f are the only ones that belong to both A and B, so $A \cap B = \{b, e, f\}$. Similarly, $A \cap C = \{a\}$. There are no elements that belong to both B and C, so $B \cap C = \{\ \}$. ◆

Example Let $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$. Then $A - B = \{a\}$ and $B - A = \{d, e\}$.

If A and B are the sets in Figure 1.9(a), then $A - B$ and $B - A$ are represented by the shaded regions in Figures 1.9(b) and 1.9(c), respectively.



(a) (b) $A - B$ (c) $B - A$

Let $A = \{1, 2, 3, 4, 5, 7\}$, $B = \{1, 3, 8, 9\}$, and $C = \{1, 3, 6, 8\}$. Then $A \cap B \cap C$ is the set of elements that belong to A, B, and C. Thus $A \cap B \cap C = \{1, 3\}$.

Example:

Suppose $\mathbf{U} = \mathbf{N} = \{1, 2, 3, \ldots\}$ is the universal set. Let

$$A = \{1, 2, 3, 4\}, \quad B = \{3, 4, 5, 6, 7\}, \quad C = \{2, 3, 8, 9\}, \quad E = \{2, 4, 6, \ldots\}$$

Find the complement of A, B, C and E

Note: A universal set, often denoted by U**,** is a set that contains **all the elements** under consideration in a particular discussion or problem.

For example, if you are working with the **set of natural numbers**, then the universal set U would include **all natural numbers**.

Solution:

$$A^C = \{5, 6, 7, \ldots\}, \quad B^C = \{1, 2, 8, 9, 10, \ldots\}, \quad E^C = \{1, 3, 5, 7, \ldots\}$$

Algebraic Properties of Set Operations

Sets under the operations of union, intersection, and complement satisfy various laws (identities) which are listed in Table ١-١.

Table 1-1 Laws of the algebra of sets

Idempotent laws:	(1a) $A \cup A = A$	(1b) $A \cap A = A$
Associative laws:	(2a) $(A \cup B) \cup C = A \cup (B \cup C)$	(2b) $(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws:	(3a) $A \cup B = B \cup A$	(3b) $A \cap B = B \cap A$
Distributive laws:	(4a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	(4b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws:	(5a) $A \cup \emptyset = A$	(5b) $A \cap \mathbf{U} = A$
	(6a) $A \cup \mathbf{U} = \mathbf{U}$	(6b) $A \cap \emptyset = \emptyset$
Involution laws:	(7) $(A^C)^C = A$	
Complement laws:	(8a) $A \cup A^C = \mathbf{U}$	(8b) $A \cap A^C = \emptyset$
	(9a) $\mathbf{U}^C = \emptyset$	(9b) $\emptyset^C = \mathbf{U}$
DeMorgan's laws:	(10a) $(A \cup B)^C = A^C \cap B^C$	(10b) $(A \cap B)^C = A^C \cup B^C$

Exercise: let U = {a, b, c, d, e, f, g, h, k}, A = {a, b, c, g}, B = {d, e, f, g}, C = {a, c, f }, and D = { f, h, k}.

١. Compute

(a) $A \cup B$ **(b)** $B \cup C$ **(c)** $A \cap C$

(d) $B \cap D$ **(e)** $(A \cup B) - C$ **(f)** $A - B$

(g) $\overline{A}$ **(h)** $A \oplus B$ **(i)** $A \oplus C$

(j) $(A \cap B) - C$

Solution:

(a) $A \cup B$ (union of A and B):

$$A \cup B = \{a, b, c, d, e, f, g\}$$

(b) $B \cup C$ (union of B and C):

$$B \cup C = \{a, c, d, e, f, g\}$$

(c) $A \cap C$ (intersection of A and C):

$$A \cap C = \{a, c\}$$

(d) $B \cap D$ (intersection of B and D):

$$B \cap D = \{f\}$$

(e) $(A \cup B) - C$ (set difference of union of A and B with C):

$$(A \cup B) - C = \{b, d, e, g\}$$

(f) $A - B$ (set difference of A and B):

$$A - B = \{a, b, c\}$$

(g) $A^{\complement}$ (complement of A):

$$A^{\complement} = U - A = \{d, e, f, h, k\}$$

(h) $A \oplus B$ (symmetric difference of A and B):

$$A \oplus B = (A - B) \cup (B - A) = \{a, b, c, d, e\}$$

(i) $A \oplus C$ (symmetric difference of A and C):

$$A \oplus C = (A - C) \cup (C - A) = \{b, d, e, g\}$$

(j) $(A \cap B) - C$ (set difference of intersection of A and B with C):

$(A \cap B) - C = \{\}$ (the result is an empty set)

Note:

The symmetric difference of sets A and C, denoted by $A \oplus C$, is defined as the set of elements that are in either A or C, but not in both.

Sequence

Introduction:

A sequence is simply a list of objects arranged in a clear order; a first element, second element, third element, and so on. The list may stop after n steps, or it may go on forever. The sequence is **finite** or **infinite**. The elements may all be different, or some may be repeated.

Example 1 The sequence 1, 0, 0, 1, 0, 1, 0, 0, 1, 1, 1 is a finite sequence with repeated items. The digit zero, for example, occurs as the second, third, fifth, seventh, and eighth elements of the sequence.

Example 2 The list 3, 8, 13, 18, 23, . . . is an infinite sequence. The three dots in the expression mean "and so on," that is, continue the pattern established by the first few elements.

Example 3 Another infinite sequence is 1, 4, 9, 16, 25, . . . , the list of the squares of all positive integers.

Example: when $C_1 = 5$, $C_n = 2C_{n-1}$ where ٢≤ n ≤ ٦ fine the sequence?

Solution

$$C_2 = 2\,C_{2-1} = 2\,C_1 = 2 * 5 = 10$$

$$C_3 = 2\,C_{3-1} = 2\,C_2 = 2 * 10 = 20$$

$$C_4 = 2\,C_{4-1} = 2\,C_3 = 2 * 20 = 40$$

$$C_5 = 2\,C_{5-1} = 2\,C_4 = 2 * 40 = 80$$

$$C_6 = 2\,C_{6-1} = 2\,C_5 = 2 * 80 = 160$$

The sequence is ٥ , ١٠ , ٢٠ ,٤٠ , ٨٠,١٨٠ and it is **finite**.

Example: define the formal by this sequence ٣, ٧ , ١١, ١٥, ١٩, ٢٣, …….

Solution: it is clear that the difference between numbers is ٤.

$$d_1 = 3 \text{ , } d_n = d_{n-1} + 4$$

Example An ordinary English word such as "sturdy" can be viewed as the finite sequence

s, t, u, r, d, y

composed of letters from the ordinary English alphabet.

Characteristic function

If A is a subset of a universal set U, the characteristic function f_A of A is defined for each x ∈ U as follows:

$$f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

Computer Representation of Sets and Subsets

Another use of characteristic functions is in **representing sets in a computer**. To represent a set in a computer, the elements of the set must be arranged in a sequence.

when a universal set U is finite, say U = {x١, x٢,..., xn}, and A is a subset of U, then the characteristic function assigns ١ **to an element that belongs to A and ٠ to an element that does not belong to A.**

Example: let U = {١,٢,٣,٤,٥,٦} , A= {١,٢} , B= {٢,٤,٦} , C={٤,٥,٦}, find $f_A(x)$, $f_B(x), f_C(x)$

Solution:

$$f_A(x) = 1 \text{ , } 1 \text{ , } 0{,}0{,}0{,}0$$

$$f_B(x) = 0 \text{ , } 1{,}0{,}1{,}0{,}1$$

$$f_C(x) = 0, 0, 0{,}1 \text{ , } 1{,}1$$

Example: let U = {a, b, e, g, h , r, s ,w} and S= {a, e, r, w} find

$F_S(x)$

Solution:

$$f_S(x) = \begin{cases} 1 & \text{if } x = a, e, r, w \\ 0 & \text{if } x = b, g, h, s. \end{cases}$$

Example: write a formula for the n^{th} term of the sequence.

a) 1, 3, 5, 7,... c) 1, 4, 7, 10, 13, 16

Solution:

a) $a_n = 2n - 1$ % This is an arithmetic sequence with a common difference of ٢

c) $a_n = 3n - 2$ % This is an arithmetic sequence with a common difference of ٣

Example: Let U = {b, d, e, g, h, k, m, n}, B = {b}, C = {d, g, m, n}, and D = {d, k, n}.

(a) What is f_B(b)? (b) What is f_C(e)?

(b) Find the sequences of length 8 that correspond to f_B, f_C, and f_D.

Solution:

$f_B(b) = b$ % This function selects elements from set B

$f_C(e)$

= is undefined % function select elements from set C. Since e is not in set

$f_B = (b, b, b, b, b, b, b, b)$
f_C =(d,m,g,n,d,m,g,m,n)

f_D = (d, k, n, d, k, n, k, n)

Homework:

Write out the first four terms (begin with n = ١) of the sequence whose general term is given.

$$c_1 = 2.5, c_n = c_{n-1} + 1.5$$

$$d_1 = -3, d_n = -2d_{n-1} + 1$$

$$e_1 = 0, e_n = e_{n-1} - 2$$

$$f_1 = 4, f_n = n \cdot f_{n-1}$$

Write a formula for the nth term of the sequence, ١, −١, ١, −١, ١, −١, ...

Matrix

A matrix is a rectangular array of numbers arranged in m horizontal rows and n vertical columns:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Example Let

$$\mathbf{A}=\begin{bmatrix}2 & 3 & 5\\ 0 & -1 & 2\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}2 & 3\\ 4 & 6\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1 & -1 & 3 & 4\end{bmatrix}$$

$$\mathbf{D}=\begin{bmatrix}-1\\ 2\\ 0\end{bmatrix},\quad \text{and}\quad \mathbf{E}=\begin{bmatrix}1 & 0 & -1\\ -1 & 2 & 3\\ 2 & 4 & 5\end{bmatrix}.$$

Then $\mathbf{A}$ is 2×3 with $a_{12}=3$ and $a_{23}=2$, $\mathbf{B}$ is 2×2 with $b_{21}=4$, $\mathbf{C}$ is 1×4, $\mathbf{D}$ is 3×1, and $\mathbf{E}$ is 3×3.

Example Each of the following is a diagonal matrix.

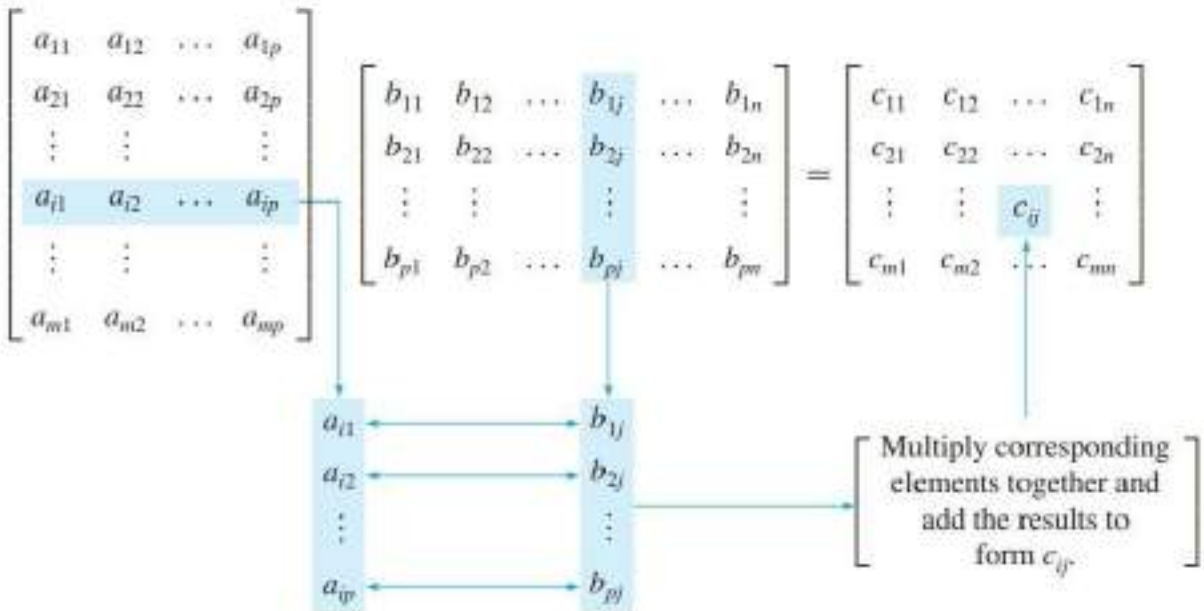
$$\mathbf{F}=\begin{bmatrix}4 & 0\\ 0 & 3\end{bmatrix},\quad \mathbf{G}=\begin{bmatrix}2 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & 5\end{bmatrix},\quad \text{and}\quad \mathbf{H}=\begin{bmatrix}0 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 6\end{bmatrix}$$

Example The following matrix gives the airline distances between the cities indicated.

$$\begin{array}{l} \\ \text{London} \\ \text{Madrid} \\ \text{New York} \\ \text{Tokyo}\end{array}\begin{array}{c}\begin{array}{cccc}\text{London} & \text{Madrid} & \text{New York} & \text{Tokyo}\end{array}\\ \begin{bmatrix}0 & 785 & 3469 & 5959\\ 785 & 0 & 3593 & 6706\\ 3469 & 3593 & 0 & 6757\\ 5959 & 6706 & 6757 & 0\end{bmatrix}\end{array}$$

Example Let $\mathbf{A}=\begin{bmatrix}3 & 4 & -1\\ 5 & 0 & -2\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}4 & 5 & 3\\ 0 & -3 & 2\end{bmatrix}$. Then

$$\mathbf{A}+\mathbf{B}=\begin{bmatrix}3+4 & 4+5 & -1+3\\ 5+0 & 0+(-3) & -2+2\end{bmatrix}=\begin{bmatrix}7 & 9 & 2\\ 5 & -3 & 0\end{bmatrix}.$$



Example Let $\mathbf{A} = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 & 1 \\ -2 & 2 \\ 5 & -3 \end{bmatrix}$. Then

$$\mathbf{AB} = \begin{bmatrix} (2)(3) + (3)(-2) + (-4)(5) & (2)(1) + (3)(2) + (-4)(-3) \\ (1)(3) + (2)(-2) + (3)(5) & (1)(1) + (2)(2) + (3)(-3) \end{bmatrix}$$
$$= \begin{bmatrix} -20 & 20 \\ 14 & -4 \end{bmatrix}.$$

Multiply matrix

Mathematical Logic:

Definition: Methods of reasoning (طرق التفكير) provides rules and techniques to determine whether an argument (البرهان) is valid.

Logic and Propositional (افتراضي)

A statement or proposition is a declarative sentence that is either true or false, but not both.

Example 1 Which of the following are statements?

(a) The earth is round.
(b) $2 + 3 = 5$
(c) Do you speak English?
(d) $3 - x = 5$
(e) Take two aspirins.
(f) The temperature on the surface of the planet Venus is 800°F.
(g) The sun will come out tomorrow.

Solution

(a) and **(b)** are statements that happen to be true.
(c) is a question, so it is not a statement.
(d) is a declarative sentence, but not a statement, since it is true or false depending on the value of x.
(e) is not a statement; it is a command.
(f) is a declarative sentence whose truth or falsity we do not know at this time; however, we can in principle determine if it is true or false, so it is a statement.
(g) is a statement since it is either true or false, but not both, although we would have to wait until tomorrow to find out if it is true or false.

Which of the following are statements?
(a) Is ٢ a positive number?
(b) x ٢ + x + ١ = ٠ (c) Study logic.
(d) There will be snow in January.
(e) If stock prices fall, then I will lose money.
Solution

(a) "Is ٢ a positive number?" - This is a question, not a statement.

(b) "x^٢ + x + ١ = ٠" - This is an equation, not a statement. It becomes a statement if you specify something about the solutions of the equation.

(c) "Study logic." - This is a command or an imperative sentence, not a statement.

(d) "There will be snow in January." - This is a statement as it makes a claim that can be either true or false depending on the weather conditions.
(e) "If stock prices fall, then I will lose money." - This is a conditional statement, expressing a relationship between two events.

Note:

In mathematics, the letters x, y, z,... often denote variables that can be replaced by real numbers, and these variables can be combined with the familiar operations +, ×, −, and ÷.

The small letters are commonly used to denote the propositional variables, such as, p, q, r, s, ….

Statements or propositional variables can be combined by logical connectives to obtain compound statements.

Compound Statements

Connective	Name	Symbol
OR	Disjunction	$\vee$
AND	Conjunction	$\wedge$
NOT	Negation	$\neg$
Implication or If-then	Implication or Conditional	$\rightarrow$
If and only if	Equivalence or Biconditional	$\leftrightarrow$

١- conjunction :Let P and Q be statements. The conjunction of P and Q, written P **^ Q , is the statement formed by joining statements P and Q using the word “and”**

Conjunction:		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for Conjunctions

Note:

^ = AND gate ,
The truth value of a proposition is true, denoted by T or ١, if it is a true proposition and false, denoted by F or ٠, if it is a false proposition. Example:

Form the conjunction of p and q for each of the following.

(a) p: It is snowing. q: I am cold.
(b) p: $2 < 3$ q: $-5 > -8$
(c) p: It is snowing. q: $3 < 5$

Solution

(a) $p \wedge q$: It is snowing and I am cold.
(b) $p \wedge q$: $2 < 3$ and $-5 > -8$
(c) $p \wedge q$: It is snowing and $3 < 5$.

٢. Disjunction

Let P and Q be statements. The disjunction of P and Q, written P v Q , the joining statements P and Q using the word "or"

The symbol v is read "OR" P vQ.

A disjunction is false only if both propositions are false as shown in the truth table.

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

Example:

Form the disjunction of p and q for each of the following.

(a) p: 2 is a positive integer q: $\sqrt{2}$ is a rational number.

(b) p: $2 + 3 \neq 5$ q: London is the capital of France.

Solution

(a) $p \vee q$: 2 is a positive integer or $\sqrt{2}$ is a rational number. Since p is true, the disjunction $p \vee q$ is true, even though q is false.

(b) $p \vee q$: $2 + 3 \neq 5$ or London is the capital of France. Since both p and q are false, $p \vee q$ is false.

٣- Negation if p is a statement, the negation of p is the statement not p, denoted by ~p or ¬P.

Truth Table

p	$\sim p$
T	F
F	T

Example:

Give the negation of the following statements:

(a) p: $2 + 3 > 1$ (b) q: It is cold.

Solution

(a) $\sim p$: $2 + 3$ is not greater than 1. That is, $\sim p$: $2 + 3 \leq 1$. Since p is true in this case, $\sim p$ is false.

(b) $\sim q$: It is not the case that it is cold. More simply, $\sim q$: It is not cold.

Example: What is the truth table for the compound proposition $\neg (p \wedge \neg q)$

Solution:

p	q	¬q	p∧¬q	¬(p∧¬q)
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

Example: Make a truth table for the statement (p ∧ q) ∨ (~p).

Solution:

Because two propositions are involved, the truth table will have ٢^٢ or ٤ rows.in the first two columns we list all possible pairs of truth values for p and q.

p	q	$p \wedge q$	$\vee$	$\sim p$
T	T	T	T	F
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T
		(1)	(3)	(2)

Exercises:

In each of the following, form the conjunction and the disjunction of p and q.

(a) p: $3 + 1 < 5$ $\qquad$ q: $7 = 3 \times 6$

(b) p: I am rich. $\qquad$ q: I am happy.

—--

In Exercises 8 and 9, find the truth value of each proposition if p and r are true and q is false.

8. **(a)** $\sim p \wedge \sim q$ **(b)** $(\sim p \vee q) \wedge r$

(c) $p \vee q \vee r$ **(d)** $\sim(p \vee q) \wedge r$

9. **(a)** $\sim p \wedge (q \vee r)$ **(b)** $p \wedge (\sim(q \vee \sim r))$

(c) $(r \wedge \sim q) \vee (p \vee r)$ **(d)** $(q \wedge r) \wedge (p \vee \sim r)$

—----

Give the negation of each of the following statements.

(a) $2 + 7 \leq 11$

(b) 2 is an even integer and 8 is an odd integer.

Lecture-٥ discrete structure , Teacher Mr. karar aljawaheri , Email : karrar.aljawaheri@gmail.com conditional statement, or implication

If p and q are statements, the compound statement “if p then q,” denoted p ⇒ q, is called a **conditional statement**, or implication. The statement p is called **hypothesis** فرضية, and the statement q is called the **conclusion** الاستنتاج.

Truth table of conditional compound

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: Determine whether each of these conditional statements is true or false.

a) If ١+١=٣ then unicorns exist.

b) If ١+١=٢ then cat can fly.

c) If ٢+٢=٤ then ١+٢=٣ **Solution:**

a) The antecedent (if part) is false (١+١ is not equal to ٣), so the entire statement is true **regardless of the** consequent (then part). Therefore, this statement is **true**.

b) The antecedent is true, and the consequent is false. In a conditional statement, if the antecedent is true and the consequent is false, the entire statement is **false**.

c) Both the antecedent and the consequent are true. In this case, when both parts of a conditional statement are true, the entire statement is true. Therefore, this statement is **true**.

Example:

Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$.

Solution

The following table is constructed using steps 1, 2, and 3 as given in Section 2.1. The numbers below the columns show the order in which they were constructed.

p	q	$p \Rightarrow q$	$\sim q$	$\sim p$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T
		(1)	(2)	(3)	(4)	(5)

BiConditional Statement, ↔

In logic and mathematics, the logical biconditional, also known as material biconditional or **equivalence** مكافئ , means the two compound statements (p called the antecedent الحالة السابقة, q, called the consequent يترتب على ذلك) are equal.

The following is a truth table for $P \leftrightarrow Q$:

P	Q	$P \leftrightarrow Q$
True	True	True
True	False	False
False	True	False
False	False	True

Example:

The English statement “If it is raining, then there are clouds is the sky” is a conditional statement. It makes sense because if the antecedent “it is raining” is true, then the consequent “there are clouds in the sky” must also be true.

Example: Determine whether these biconditionals are true or false. a) ٢+٢=٤ if and only if ١+١ =٢ : T

b) ١+١=٢ if and only if ٢+٣=٤ : F

c) ١+١ =٣ if and only if monkeys can fly. T

d) ٠>١ if and only if ٢>١ : F

Exercise:

Chapter-٣ relations

Relationships between people, numbers, sets, and many other entities can be formalized in the idea of a binary relation.

Binary relation:

There are many relations in mathematics :"less than" , "is parallel to ","is a subset of", etc. These relations consider the existence or nonexistence of a certain connection between pairs of objects taken in a definite order.

Ordered Pairs:

In sets, the order of elements is not important. For example, the sets {٢, ٣} and {٣, ٢} are equal to each other. However, there are many situation in mathematics where the order of elements is essential. So, for example, the pairs of numbers with coordinates (٢, ٣) and (٣, ٢) represent different points on the plane. This leads to the concept of ordered pairs. An ordered pair consists of two elements, say a and b, in which one of them, say as the first element and the other as the second element. An ordered pair is denoted by (a,b) .

Remark:

1- . An ordered pair (a,b) can be defined by (a,b) = {{A},{a,b}}

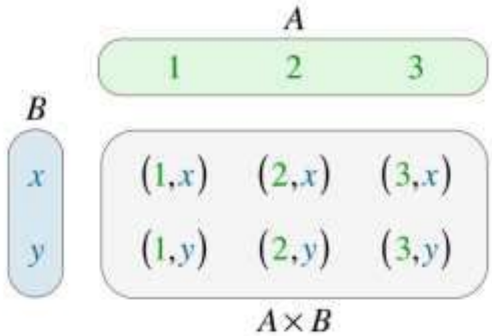
2- Ordered pairs can have the same first and second elements such as (1,1) , (4,4).

3- if a=b than (a,b)={{a}}

4- if a dot equal to b then (a,b) do not equal to (b,a)

Product sets:

Product Set: Let A and B be two sets. The product set of A and B consists of all ordered pairs (a, b) where $a \in A$ and $b \in B$. It is denoted by $A \times B$ which reads "$A\ cross\ B$". i.e. $A \times B = \{(a, b), a \in A, b \in B\}$.



Example:

1. $A = \{1, 2, 3\}$ and $B = \{a, b\}$. Then the product set $A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$.

2. Let $w = \{s, t\}$. Then $w.w = \{(s, s), (s, t), (t, s), (t, t)\}$.

Example: Let A = {1,2} and B = {a ,b ,c} then

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$

Also, $A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

Remark:

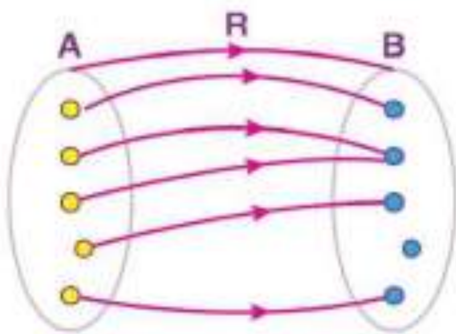
١- If set has n elements and set has m elements then the product set AXB has n.m elements. ٢- In general A X B not equal to B X A.

٣- If A is any set and $B = \emptyset$ then $A \times B = \emptyset$.

Relation:

A **relation** is a subset of an order product. Basically, a relation is a rule that “relates” an element from one set to an element from another set.

$R = \{(x,y): x \in A \text{ and } y \in B\}$



Examples:

1. Let $A = \{1,2,3\}$ and $B = \{a,b\}$. Then $R = \{(1,a),(1,b),(3,a)\}$ is a relation from A to B. Furthermore, $1\ R\ a, 2 \not R\ b, 3\ Ra, 3 \not R\ b$.

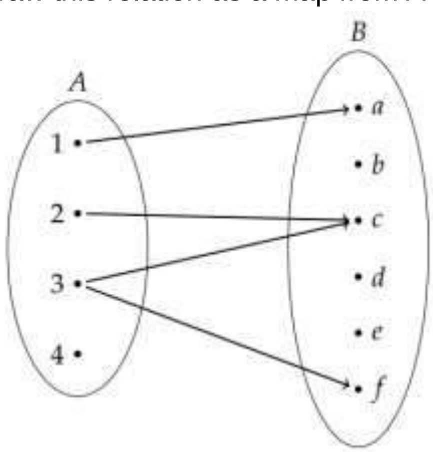
2. Let $w = \{a,b,c\}$. then $R = \{(a,b),(a,c),(c,c),(c,b)\}$ is a relation in w. Moreover, $a \not R\ a,\ b \not R\ a,\ c\ R\ c,\ a\ R\ b$.

Example-٢ Define relation R as a positive integer and twice its value.

$$R = \{(1,2),(2,4),(3,6),\dots\}$$

Example ٣:

Suppose A = {١, ٢, ٣, ٤}, B = {a, b, c, d,e, f }, and relation R = {(١, a),(٢, c),(٣, f),(٣, c)}. We can draw this relation as a map from A to B, given in Figure below.



Reflexive relation: Let R be a relation in a set A, i.e. Let R be a subset of $A \times A$. then R is called a reflexive relation if for every $a \in A$, $(a,a) \in R$. In other words, R is reflexive if every element in A is related to its.

Example:

1. Let $V = \{1,2,3,4\}$ and $R = \{(1,1),(2,4),(3,3),(4,1),(4,4)\}$. then R is not reflexive since $(2,2)$ dose not belong to R. Notice that all ordered pairs (a,a) must belong to R in order for R to be reflexive.

Symmetric relation: Let R be a relation in a set A.Then R is called a symmetric relation if

(a,b) ∈ R and (b,a) ∈ R.

Example:

Let S= {١,٢,٣} and let R={(١,٣) , (٤,٢),(٢,٤),(٢,٣),(٣,١)} then R is not symmetric relation since (٢,٣) does not exist.

Example:

Let A={١,٢,٣} Determine whether the relation is reflexive, symmetric.

1. $R = \{(1,1),(1,2),(2,1),(2,2),(3,3)\}$.

2. $R = \{(1,3),(1,1),(3,1),(1,2),(3,3)\}$.

Solution:

1. Reflexive, symmetric
2. Not reflexive, not symmetric